Sensitivity Analysis of an Agent-Based Model of Culture's Consequences for Trade

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Abstract This paper describes the analysis of an agent-based model's sensitivity to changes in parameters that describe the agents' cultural background, relational parameters, and parameters of the decision functions. As agent-based models may be very sensitive to small changes in parameter values, it is of the essence to know for which changes the model is most sensitive. A long-standing metamodeling-based approach of sensitivity analysis is applied to the agent-based model. The analysis is differentiated for homogeneous and heterogeneous agent populations. Intrinsic stochastic effects of the agent-based model are taken into account. The paper describes how an appropriate regression model has been selected and analyses the parameter's variance contributions in general and in specific cultural settings.

1 Introduction

Agent-based models are known to be very sensitive to parameter changes in some ranges of the parameter space. Small changes in parameter values may have dramatic consequences for the state of the system, while changes in other parts of the parameter space have little effect. This property of multi-agent systems is usually referred to as non-linearity. It is not just a property of agent-based models. It is a general property of complex systems such as ecosystems, climate, and the economy.

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Non-linearity may lead to abrupt changes in the state of systems, and this property invites to the application of agent-based models to simulate non-linear effects such as catastrophic events in evolution or economics [3, 12]. We may conclude that non-linearity is not a bad property of agent-based models. It is a general property of complex systems that complicates the work of modelers of such system.

In general it is considered good modeling practice to perform sensitivity analysis as a part of model verification [16]. In the case of agent-based models, two reasons urge to perform extensive sensitivity analysis: great uncertainty about actual values of model parameters and non-linearity. For instance, in the model discussed by Kirman [11], a tipping point between loyalty to trade partners and shopping behavior exists, depending on the value of the loyalty parameter β. If one wants to apply such a model in multi-agent models of markets, the agents have to be configured with actual values for β. For some range of low values of β, the value will not have an effect on the shopping behavior of a single agent. Around some critical value of β, there is an abrupt change, and there is a relatively small range of increasing loyalty. For a large range of higher values for β, the behavior is invariably loyal. As a result, depending on the actual distribution of β in the agent population, the efficiency of an artificial market may be very sensitive to small changes in the distribution of β, or may be rather insensitive to even larger changes. However, it is hard to predict the actual distribution of β for a particular context.

Because of the combination of non-linearity and uncertainty about parameter value distributions, extensive sensitivity analysis is a sine qua none for research with agent-based models. Before a conclusion can be drawn on the basis of an agent-based model, the modeler must search for the regions in parameter space where stable, maybe inactive, states of the system occur and where the model is insensitive to parameter changes, regions where tipping points occur and system behavior changes dramatically in case of small parameter changes, and regions where the system is more or less proportionally sensitive to parameter changes.

This paper presents the approach and results of extensive sensitivity analyses of a model of culture's effects on international trade. The multi-agent model is based on a model of a trade game that allows for experimental data collection on trust in supply chains with asymmetric quality information [9]. The model is based on transaction cost economics [19]. The agents' activities cover partner search, negotiation, and, if negotiation leads to a contract, truthful delivery or opportunism, taking advantage of the information asymmetry. Their counterparts may either trust the deliveries, or incur cost to monitor and enforce contract fulfillment. The agent model of Jonker et al. [9] has been refined and extended with differentiation of agent behavior according to cultural background [5, 6, 7]. For this purpose, rules were formulated for adaptation of default model parameters based on Hofstede's five dimensions of culture [4].

Sensitivity analysis is performed on the extended model. A systematic sensitivity analysis can serve several purposes: improve the understanding and reliability of model results; reveal effects of parameter variations; guide simplification and refinement of the model [15]. This paper focuses on the effects of parameter variation. The following are the main questions for the sensitivity analysis.

1. Which areas in parameter space result in realistic behavior?
2. Which parameters have significant effects for which outputs?
3. Which interactions between culture and other parameters are important?
4. Are the answers different between aggregate and individual level?

Sensitivity analysis basically consists of a statistical analysis of the effect of input variations on model outputs. Richiardi et al. [15] identify types of variations of inputs. These types can be grouped into (I) variations of random seed and noise level, (II) variations of parameter values, (III) variations of the model, e.g. agent's decision functions, data aggregation, time scale and sample size. The present paper focuses on the first two groups of variation. It studies the effect of intrinsic variation caused by the stochastic nature of the model and the effect of external variation of model parameters and of culture. The sensitivity analysis approach is based on Jansen et al. [8] and Saltelli et al. [17], applying two principles:

1. meta modeling of results of parameter sets drawn at random from the joint distribution;
2. analysis of contributions of Top Marginal Variance (TMV) and Bottom Marginal Variance (BMV) of individual parameters or groups of parameters to the variance explained by the meta model.

Section 2 of this paper introduces the model and the parameters taken into account. Section 3 presents the approach of sensitivity analysis and discusses the special issues with respect to unit of analysis (system vs. individual) and heterogeneity. Section 4 presents results for some observable statistics at system and individual level. Section 5 concludes the paper with an evaluation the applied method.

2 Trading Agents with Cultural Background

The model analyzed in this paper simulates trading agents operating in a game [9]. The agents may trade with each other, are free to select or refuse a partner, negotiate or quit negotiation if they do not expect a satisfactory conclusion, and, in case of successful negotiation, exchange a commodity. The special thing about the game is that commodities have high or low quality and that the seller is informed about the quality, which is invisible for the buyer. A buyer can either trust a delivery or (at the cost of a fee) offer it to the tracing agency that reveals the real quality and in case of deceit punishes the deceiver by a fine. Another option for the buyer is to have the seller trace the commodity in advance and add the tracing report as a quality certificate. The tracing fee for sellers is lower than it is for buyers. The strategies a buyer can chose are: (1) buy low quality (no risk), (2) trust, (3) require certification, (4) trace random samples, or, (5) in addition to random tracing, negotiate that some refund will be made in case quality turns out to be non-compliant.

Details of the models of the agents' activities and the effects of culture have been described in earlier papers [5, 6, 7]. For each of these activities, a model of the agents' decisions is selected from social sciences or artificial intelligence literature.

For instance, for partner selection, the model of Weisbuch et al. [18] is used; for negotiation Jonker and Treur's ABMP architecture [10] is selected. The decision models' parameters included in the sensitivity analysis are listed in Table 1.

Table 1 Trading agent's activities and model parameters and variables that are adjusted according to an agents' cultural background [5, 6, 7]; the table also specifies the value range considered in the sensitivity analysis

Activity	Parameter or variable	Value range
Partner selection	Loyalty	0.5...1.5
	Learning	0.001...0.999
	Preference (initial value)	0.001...0.999
Negotiation	Concession factor	0.001...0.999
	Negotiation speed	0.001...0.999
	Impatience	0.001...0.999
	Quality preference	0.001...0.2
	Risk aversion	0.001...0.2
Deceit and trust	Minimal honesty	0.001...0.999
	Honesty decay factor	0.001...0.999
	Trust (initial value)	0.001...0.999
Belief update	Negative update factor	0.001...0.999
	Endowment factor	0.001...0.999

In the agent model the decision functions are influenced by a set of rules that take Hofstede's cultural dimensions and some culturally relevant relational characteristics into account [5, 6, 7]. The indices of the cultural dimensions are:

- PDI (power distance);
- UAI (uncertainty avoidance);
- IDV (individualism);
- MAS (masculinity);
- LTO (long-term orientation).

The relational characteristics taken into account are group distance (i.e. absence of common group membership) and societal status of the agent and of its partner. Cultural indices and relational characteristics are represented as real values in the range [0...1]. For the sensitivity analysis they are drawn from the range [0.001...0.999].

3 Sensitivity Analysis Approach

The sensitivity analysis reported in this paper is regression-based: a meta model in terms of the input parameters is fitted to an output variable. The output is produced by simulation runs using input parameter sets generated by Monte Carlo sampling. Monte Carlo sampling of the parameter sets aims to cover the range of all parameters efficiently and to avoid multicollinearity.

The relative importance of individual input parameters on output variables is assessed by decomposition of the variance of the output variable. The key issue in this approach is to find a regression model that can serve as a basis for decomposition of variance. Any type of regression may be applied, e.g. linear regression including polynomial and interaction terms [14] or regression with smoothing splines [2] as a form of nonparametric regression, as long as it explains a great deal (preferably at least 90%) of the output variance.

Jansen et al. [8] define the top marginal variance (TMV) of an input as the variance reduction that would occur if the input would become fully known. The bottom marginal variance (BMV) is the variance that the meta model can not explain without the input parameter. TMV and BMV of an input variable are equal if and only if that variable is not correlated with any other variable. Comparison of TMV and BMV can be used to check for multicollineairty unless interaction-terms are important. If interaction-terms are taken into account in the regression model, the BMV is defined as the variance that cannot be explained without the input parameter and all interaction terms including this parameter.

In this sensitivity analysis three sources of variance are studied:

1. cultural and relational factors, used to adapt the decision making to culture,
2. the default values of the parameters mentioned in Table 1,
3. stochastic effects caused by variation of random seed.

The approach proposed by Jansen et al. [8] was developed for equation-based models, in which there is a single level of aggregation. When analyzing multi-agent systems, the unit of analysis has to be decided: system performance at aggregate level or individual agent performance. The present study observes ouputs at aggregated level for simulations with homogeneous agent populations and at individual agent level for simulations with heterogeneous populations.

Data generation proceeds as follows. The first step is to draw input parameters sets from the joint distribution of all model parameters. As the goal is to study the effects of parameter variation and there is no accurate information on actual parameter distributions, we draw values at random from uncorrelated uniform distributions, ranging as indicated in section 2. The resulting parameters sets are used to initialize trading agents for simulation runs. In order to analyze intrinsic stochastic effects, model runs are repeated with equal parameter sets but different random seed.

The following outputs are observed:

- number of transactions;
- number of failed negotiations;
- average duration (number of rounds per negotiation)
- number of high quality transactions;
- number of deceitful transactions;
- number of traces requested;
- number of fines issued by the tracing agency;
- loyalty, measured as standard deviation of transactions per potential partner.

All statistical analyses were performed with GenStat 12th Edition (VSN International Ltd., Hemel Hempstead, Hertfortshire). Sensitivity analyses was performed with USAGE 2.0, a collection of GenStat algorithms for sensitivity and uncertainty analysis [1].

4 Results

This section presents results of the sensitivity analysis of simulations with the multi-agent model. All simulations were run with a population of 8 supplier agents and 8 customer agents. The agents were free to select or refuse a trade partner, negotiate and quit negotiations or accept an offer, and deliver truthfully or defect. The simulations ran for 100 time steps. The maximum number of transactions that can practically occur in such a run is between 160 and 180.

For the first series of simulations, parameters sets are drawn at random for configuration of homogeneous agents per run. Cultural indices, relational factors, and the default model parameters referred to in Table 1 are all drawn independently. For each parameter set the model was run 15 times with different random seed, in order to estimate the variance introduced by intrinsic stochastic effects. Statistics are collected at aggregate level. For 627 out of 1000 generated parameter sets the median of the number of transactions equaled zero over 15 replications.

4.1 Probability that transactions occur

A logistic regression model [13] was used to investigate which parameters or combination of parameters (interaction) were of significant influence on the probability whether or not transactions occurred (binary data: median equals zero or median greater than zero).

A first exploration revealed that concession factor γ is the most dominant parameter to predict the occurrence of tranaction: from 20% for low values of γ to 60% for high values.

Interactions between parameters appeared to play an important role. Starting from a logistic regression model containing all main effects, significant interactions ($p < 0.05$) have been added by forward selection. Table 2 presents the coefficients for the main effects and the significant interactions in the model.

The parameters that have significant effect without interactions are PDI, impatience, and risk avoidance. The probabilities that transactions occur are:

- 0.2789 for $PDI = 0.01$; 0.4025 for $PDI = 0.99$;
- 0.3949 for $\iota = 0.01$; 0.2839 for $\iota = 0.99$;
- 0.4075 for $w_r = 0.01$; 0.2716 for $w_r = 0.20$.

Table 2 Coefficients for main effects (left hand side) and interactions (right hand side) in the logistic regression model of the probility that transactions occur in a simulation run

Parameter	Symbol	Coefficient	Interaction	Coefficient
Power distance	PDI^*	0.566	$\lvert s_a - s_b \rvert \cdot \gamma$	-4.39
Uncertainty avoidance	UAI^*	-0.122	$\bar{s} \cdot \gamma$	-4.43
Individualism	IDV^*	2.015	$MAS^* \cdot v$	-2.581
Masculinity	MAS^*	2.300	$LTO^* \cdot \gamma$	3.134
Long-term orientation	LTO^*	3.02	$LTO^* \cdot MAS^*$	-3.108
Group Distance	D	0.211	$\bar{s} \cdot t_0$	-4.37
Mean status	$\bar{s}$	7.29	$\bar{s} \cdot LTO^*$	-3.38
Status difference	$\lvert s_a - s_b \rvert$	0.762	$\lvert s_a - s_b \rvert \cdot LTO^*$	2.69
Loyalty	B	0.276	$LTO^* \cdot w_q$	-14.18
Learning	C	-0.454	$IDV^* \cdot w_q$	-12.13
Initial preference	J_0	-1.948	$D \cdot LTO^*$	-2.426
Concession factor	γ	4.42	$\lvert s_a - s_b \rvert \cdot J_0$	2.99
Negotiation speed	v	1.015	$J_0 \cdot w_q$	9.59
Impatience	ι	-0.509	$IDV^* \cdot h$	-1.884
Quality preference	w_q	3.27	$e.f$	-2.934
Risk aversion	w_r	-3.22		
Minimal honesty	h	0.296		
Honesty decay factor	f	1.265		
Initial trust	t_0	2.919		
Negative update factor	u_-	-0.062		
Endowment factor	e	1.231		

For parameters that have significant interactions, probabilities can only be predicted if the interactions are taken into account. For instance, the effect of MAS can be predicted in interaction with LTO and negotiation speed v. Table 3 shows that the effect of MAS is great if LTO and negotiation speed are both high or both low.

Table 3 Prediction of the probability that transactions occur with different values of MAS in interaction with LTO and negotiation speed v

LTO^*	MAS^*	$v = 0.01$	$v = 0.99$
0.01	0.01	0.1889	0.3805
	0.99	0.6774	0.3170
0.99	0.01	0.4130	0.6497
	0.99	0.2427	0.0662

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From study of interaction tables like Table 3, it is concluded that transactions are unlikely to occur ($p < 0.20$ for extreme values of the parameters) if

- group distance and LTO are both high
- status difference and concession factor are both low
- MAS, LTO, and negotiation speed are all high or all low
- status difference and intitial trust are both low
- IDV, LTO, and quality preference are all high
- status difference is low and initial partner preference is high
- initial partner preference is low and quality preference is high
- IDV and minimal honesty are both high
- honesty decay factor and endowment factor are both high

4.2 Sensitivity analysis

For analysis of the relations between parameters values and outputs a set of 1000 simulations with at least 16 successful transactions is generated. Parameter sets are randomly drawn and used for homogenous configuration of agents in simulation runs, until 1000 runs have produced at least 16 transactions. For each of the 1000 selected parameter sets 15 replications are run. The replications are used for analysis of the variance between parameter sets versus the variance caused by stochatic effects in the replications (Table 4). The percentages are small. the variation between simulations is dominantly caused by parameter variation.

Table 4 Mean variance in replications as percentage of total variance

observed output	% variance	observed output	% variance
number of transactions	1.60	number of deceitful transactions	8.63
number of failed negotiations	3.95	number of traces	7.71
average duration of negotiations	5.32	number of fines	13.75
number of high quality transactions	4.68	average loyalty	5.81

The mean values of outputs of 15 simulations per parameter set are used for analysis. As an example we treat the analysis of the number of transactions. Straighforward sensitivity analysis based on a smoothing spline with two degrees of freedom results in 61.3% of the variance accounted for. For a few parameters the difference between the top and bottom marginal variance is substantial. This can only be due to correlations between parameters (caused by the selection proces). Correlation coefficients are small, see Table 5. Therefore, correlations are not further analyzed.

Since 39% of the variation is not explained, several other models are tried, including smoothing splines with 5 degrees of freedom (63.1%), polynomial models, models taking second and third level interactions into account, and log transformations on output and on both input and output. The models using log tranformations perform worse than models with polynomial and interaction terms (74.1% and 44.4%, respectively).

Table 5 Parameters having correlation coefficients of 0.10 or more

Parameter	Parameter	Correlation coefficient
Concession factor	Group distance	0.11
	Mean status	-0.14
	Quality preference	0.13
	IDV	0.11
	LTO	0.15
Mean status	LTO	-0.11
Negotiation speed	Quality preference	0.16

The best fit is obtained with a model including quadratic terms and 33 two and three factor interaction terms, that explained 80.7% of the variation. For efficiency the sensitivity analysis is performed with a model with quadratic terms and two factor interactions that explains 79.5%. All parameter combinations in the three factor interactions are also represented as two-factor interactions in the latter model.

For all variables and their interactions both linear and quadratic terms are taken into account (comparable to a smoothing spline with df=2). The result is a model with 30 two-factor, forwardly selected, interaction terms explaining 79.5% of the variation. The interest is not in the model but in its use for gaining insight in the sensitivity of the multi-agent model. Based on this model the bottom marginal variance is calculated for each parameter by leaving this variable and all interaction-terms involving this variable, out of the model. Table 6 presents top and bottom marginal variances.

Table 6 Top Marginal Variance and Bottom Marginal variance of parameters as percentage of the total variance of the number of transactions

Parameter	TMV(%)	BMV(%)	Parameter	TMV(%)	BMV(%)
Index of culture			Loyalty parameter	0.0	0.0
– PDI	0.0	0.6	Loyalty decay factor	0.0	0.1
– IDV	0.2	5.3	Concession factor	9.1	25.0
– UAI	0.8	3.4	Negotiation speed	31.8	39.3
– LTO	0.7	6.8	Impatience	0.6	2.5
– MAS	2.0	7.7	Quality preference	1.5	0.7
Group distance	2.7	6.8	Risk avoidance	0.0	3.0
Mean status	0.3	5.1	Negative update factor	0.9	0.3
Status difference	0.0	1.9	Endowment factor	0.1	0.0
Initial trust	2.7	6.3	Minimal honesty	0.0	0.0
Initial partner preference	1.2	3.0	Honesty decay factor	0.0	0.0

Variation in the number of transactions is for 32% due to variation in negotion speed. Some other inputvariables interact with negotiation speed, resulting in a bottom marginal variance of 39%. This means that without good information about negotiation speed 39% of the variation in the number of transactions will remain.

The differences between TMV and BMV of culture and relational factors indicate that these parameters largely have their effect in interactions.

The model developed for the number of transactions cannot be applied for sensitivity analysis of the other output variables. The percentage of variation explained is unsatisfactory. Sensitivity analyses for other outputs have been carried out straightforwardly using smoothing splines (df=2), also resulting in unsatisfactory explanation of variation. The results indicate that modeling steps as applied for the number of transactions need to be followed for each output individually. The parameters contributing at least 10% to output variation according to the sensitivity analysis with smoothing splines (df=2) are given below for each output variable.

- negotiation failure: negotiation speed, concession factor, impatience
- negotiation duration: negotiation speed, UAI, MAS
- quality: intitial trust, LTO, quality preference
- deceit: initial trust, MAS
- tracing: MAS
- fines: MAS
- loyalty: initial partner preference, negotiation speed

4.3 Differences between cultures

As found in the preceeding subsections there are many interactions between parameters. To further analyse the interaction with culture, sensitivity analyses are performed for 62 actual national cultures. 1000 simulations are run for each culture, each simulation with a ramdomly drawn parameter set that is used to configure a homogeneous agent population. Sensitivity analysis is performed on the simulations that result in at least one transaction. The purpose of this step is to focus sensitivity analysis on the parameters in Table 1 and relational characteristics, and to find differences in sensitivity between cultures.

The number of simulations resulting in one or more transactions ranges from 228 through 490 across cultures, with mean 403. Taking only the runs with a positive number of transactions into account, three different models were fitted per culture (the minimum and maximum percentage of variation explained across cultures is given in parentheses):

- with all 16 parameters linear in the model (minimum 70.7%, maximum 81.2%),
- with all 16 parameters as a spline with 3 degrees of freedom in the model (minimum 73.3%, maximum 83.4%),
- with all 16 parameters and their first order interactions in the model (minimum 83.4%, maximum 89.8%).

No strong nonlierar effects seem to occur in the analyses per culture. Interactions between parameters are present. Table 7 presents some results from the analyses per culture. The sensitivity for parameter changes varies widely across cultures.

Table 7 Mean Top Marginal Variance values (of 62 countries) and data for the countries that have the maximum TMV score for a parameter

national culture	group distance	mean status	initial trust	partner pref.	conces. factor	negot. speed	quality pref.	risk avoid.
Mean (n=62)	4.1	1.0	3.4	1.8	24.8	30.2	0.5	1.6
Indonesia	16.9	0.1	0.2	0.0	11.6	40.9	0.0	0.0
Morocco	0.7	8.7	3.3	4.7	17.5	37.9	0.0	0.0
Hungary	0.0	0.0	11.5	1.9	37.6	1.4	2.4	11.5
Uruguay	4.9	1.7	8.5	5.5	23.4	24.1	0.0	0.0
Netherlands	0.7	0.0	1.6	0.3	46.2	28.8	0.0	2.1
Iran	1.3	3.1	0.9	0.4	10.3	56.2	0.6	0.0
Austria	0.0	0.0	3.8	2.6	27.1	11.5	4.8	6.9
Japan	0.5	0.0	8.3	1.6	30.8	0.9	0.1	15.8

4.4 Aggregate and individual level

To perform sensitivity analysis at agent level in heterogeneous agent populations, parameter sets for 4000 simulations are drawn. First 4000 sets of cultural indices, group distance, and status data are drawn. For each simulation run, all agents are configured with equal culture. This resticts partner selection to partners with equal cultural background. For each agent the other parameters are randomly drawn, resulting in a sample of 32000 suppliers and 32000 customers.

A sensitivity analysis could not be completed. Too low levels of explained variation were obtained: for the number of transaction the explained variation was for supplier agents 48.5% with linear fit and with 51.3% smoothing splines; for customer agents 37.6% with linear fit and 38.6% with smoothing splines. However, an interesting result was obtained. The pattern of marginal variance of suppliers matches the pattern found at aggregate level, with negotiation speed and concession factor as dominant parameters. The pattern of marginal variance of customers is very different, with relational characteristics explaining most of the variation. The information asymmetry explains this difference: trust is relevant only for customers.

5 Conclusion

Through sensitivity analysis insight can be gained into the properties of a model.

For exploration of the regions where realistic behavior occurs, logistic regression can be used and probabilities of realistic behavior can be predicted with the model.

Parameters that have significant effects can be identified through metamodeling, even for complex systems. However, the analysis is not straightforward.

The interactions between culture and other parameters are the main cause of the model's complexity. When keeping culture constant, straightforward methods for sensitivity analysis can be applied. Results differ considerably across cultures.

Sensitivity of individual agents can differ considerably from aggregate level sensitivity. However, a method for individual agents has to be developed.

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